



Exact Path

Research-Based Development of the Middle School Skills Framework

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Executive Summary

A systematic and research-driven approach underpins the development of Edmentum’s Middle School Skills Framework for Exact Path, with a focus on Math and English Language Arts (ELA). This document highlights the key decisions, methodologies, and strategies employed to create a targeted intervention designed to address the unique needs of middle school students. The framework is crafted to ensure readiness for high school, particularly Algebra 1 and English 9, and is grounded in educational research, customer insights, and alignment with State and National Curriculum Standards.

This white paper will detail the approach the Edmentum Learning Design (LD) team took in the development of the Framework and show how it is rooted in the latest educational research and best practices. For mathematics, LD grounded recommendations in Algebra Readiness research, ensuring students become equipped with the necessary foundational skills to succeed in higher-level math. In English Language Arts, LD aligned the approach to the Science of Reading, focusing on evidence-based literacy instruction for older learners. Together, these frameworks create a coherent, research-backed progression of skills designed to address learning gaps and promote long-term success for middle school students as they prepare for high school and beyond.

Introduction

Research consistently shows that many high school students lack the foundational skills necessary to succeed in key subjects, particularly algebra and English. The National Assessment of Educational Progress (NAEP) provides insights into the proficiency levels of high school students in key subjects and has consistently shown that a majority of students do not demonstrate solid academic performance in reading or math. The most recent data ([NAEP, 2024](#)) confirm this with only 31% of eighth-grade students performing at or above the “proficient” level in reading and 28% of eighth graders achieving proficiency in math. These scores highlight significant gaps in foundational skills among middle school students. In response, the Exact Path Middle School Skills Framework was created, based on a robust foundation of academic research and real-world feedback from educators, to prepare students in the foundational skills needed to close these gaps.

To develop this new Exact Path Middle School Skills Framework, LD identified 30 essential skills per subject at each grade level (6–8) that address the most pressing educational challenges and provide students with the necessary intervention for success in Algebra I and English 9 with a goal of high alignment to standards. LD also considered the six learning design principles foundational to the development of Exact Path:



Adaptive, Individualized Learning

Instruction is tailored to each student's unique needs by adjusting the level, sequence, and pace of learning. Exact Path provides a personalized learning path that evolves as students demonstrate progress or require additional support.

Explicit and Adaptive Instruction

Complex skills are broken down into manageable steps using research-based strategies that reduce cognitive load and guide students toward independent mastery.

Active, Engaging Learning

This student-centered approach captures the unique needs of middle school learners by encouraging interaction with content through dynamic visuals, hands-on manipulatives, and contextual examples that connect to their interests and real-world experiences.



Assessment-Driven Mastery Learning

This skill-building approach ensures students master critical skills through diagnostic assessments, targeted practice, and progress checks. By focusing on areas needing reinforcement, it maximizes learning efficiency and fosters individualized student growth.

Deliberate Practice and Immediate Feedback

Structured practice aligned to clear skill statements ensures students develop mastery over time. Immediate feedback reinforces understanding and helps students correct errors in real time.

Subject-Area Best Practices

Learners build deep understanding through research-backed instructional strategies tailored to each subject area.



By integrating these learning design principles, Edmentum's Exact Path Middle School Skills Framework builds a pathway of progressively challenging skills. Lessons feature clear objectives, engaging materials, and scaffolded supports to help students succeed at every stage of their learning journey. This deliberate alignment ensures that the framework addresses the specific needs of middle school learners, ensuring that students receive the support they need at every stage.

Learning within the Zone of Proximal Development (ZPD) requires strategic supports that allow students to tackle tasks just beyond their current abilities without becoming frustrated or disengaged (Vygotsky, 1978). Scaffolding is essential in this process, providing temporary structures such as guided examples, step-by-step instructions, and targeted feedback to help students bridge the gap between what they know and what they are capable of learning.

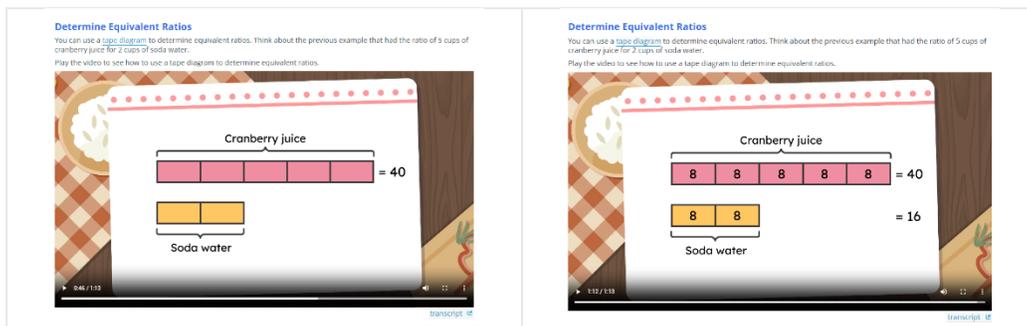


Figure 1. Grade 6 Math: Fractions and Ratios - Ratios and Ratio Language

Exact Path incorporates these scaffolding techniques, alongside explicit instruction, to ensure that middle school students can engage with increasingly complex concepts. Supporting students in developing these skills progressively supports research by Guthrie et al. (2013), who found that scaffolded instruction significantly improves comprehension and motivation among young adolescents. By gradually removing supports as proficiency grows, the framework fosters independence and mastery while maintaining an optimal level of challenge. Hattie's (2023) research on visible learning underscores that students make the most progress when they understand their learning path, receive structured support, and gradually take ownership of their learning.

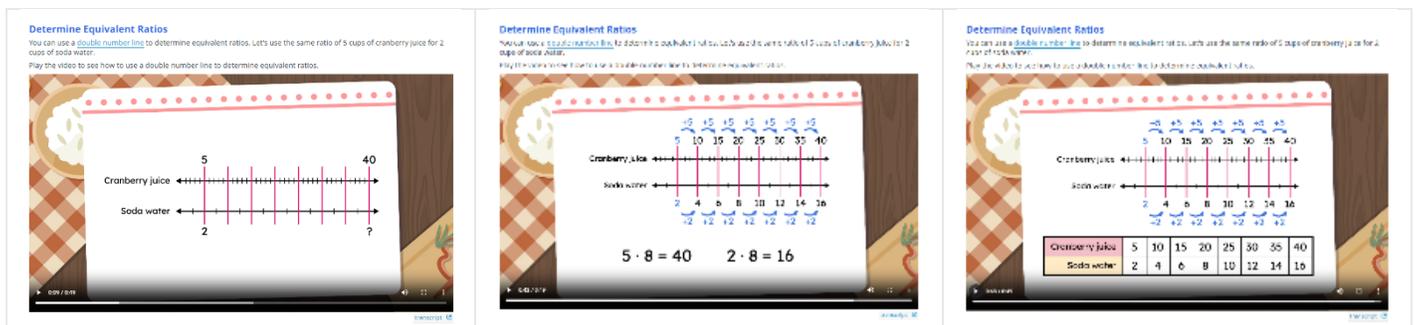


Figure 2. Grade 6 Math: Fractions and Ratios - Ratios and Ratio Language



This structured approach to teaching and learning lays the groundwork for a cohesive middle school learning progression, where each stage is carefully designed to build upon prior knowledge and skills. By aligning instructional strategies with students' developmental needs, the framework ensures a seamless progression from foundational concepts to more advanced applications. Central to this progression is the backward design methodology, which ensures that every step in the instructional process is purposefully aligned with learning goals, lesson objectives, and educational standards. This alignment fosters deeper understanding and equips students with the tools they need to succeed at each stage of their Exact Path experience. Lessons are built around clear goals and expected outcomes, with skills and concepts sequenced logically to support cumulative learning, ensuring alignment between instruction and assessments to promote mastery and reduce cognitive overload (Archer & Hughes, 2011).

Using Roots and Affixes as Clues

Use the strategy to determine the meaning of another unknown word. Read these sentences from a science textbook.

Satellites transmit data about the Earth's climate to scientists around the world. This information helps them understand weather patterns and predict future climate changes.

Let's start with the questions in steps 1 and 2 of the [Combining Word Parts](#) resource. Think to yourself: What word parts do I know in "transmit"? The word has the prefix "trans-" and the root "mit." There is no suffix. What do these word parts mean? "Trans-" means "across," and "mit" means "to send."



Figure 3. Grade 6 Reading: Roots and Affixes

The Exact Path Middle School Skills Framework is designed to support a mastery learning model, providing students with the necessary pathways to build proficiency in each skill before advancing to more complex concepts. The framework connects current learning to foundational skills through a remediation pathway that revisits and scaffolds earlier aligned skills when students encounter difficulties. This is how Exact Path helps close gaps that may impede the student's performance. In addition to the structure of the students' pathways, formative assessments are embedded throughout the instruction, allowing teachers to monitor progress and implement targeted interventions. These assessments align with research by Safer and Fleischman (2005), which emphasizes that regular feedback and progress monitoring are critical for closing learning gaps and achieving mastery.

In summary, the Exact Path Middle School Skills Framework was intentionally designed to create a pedagogical approach to curriculum and structure rooted in six fundamental principles of learning design. These principles were applied by the creation of skills pathways that break learning into singular,



targeted skills that can be scaffolded to meet learning needs of students for both learning acceleration and remediation. The rationale behind these skills and strategies is grounded in robust educational research and tailored to meet the unique needs of middle school learners. By aligning these principles with the developmental and academic demands of this age group, Exact Path ensures that students are well-prepared for high school success.

Middle School Learning

According to the Association for Middle Level Education (AMLE), key strategies to improve student achievement in middle school include: creating a relevant and engaging curriculum, fostering positive relationships with students, utilizing differentiated instruction, incorporating collaborative learning, providing frequent feedback, addressing the unique developmental needs of young adolescents, and ensuring strong teacher support and professional development—all while focusing on building a positive school climate that supports student well-being and academic growth.

While Edmentum addresses and supports all of these aspects in the product and its implementation, the goal for the Exact Path Middle School Skills Framework was to design and develop a clear and cohesive learning pathway that:

- addresses the unique developmental needs of young adolescents
- utilizes differentiated instruction to address diverse learning needs
- enables research-based curriculum development.

Middle school learners are navigating a developmental period characterized by rapid cognitive, personal, and relational changes that significantly affect how they learn and engage. According to Immordino-Yang et al. (2019), brain development during this stage is highly responsive to environmental and social-interpersonal interactions, emphasizing the importance of creating learning environments that integrate a whole brain learning approach. This period offers a prime opportunity for educators to implement practices that align with these developmental needs, such as fostering a sense of safety and belonging, which enhances students' ability to process and retain new information.

Middle school adolescents represent a distinct age group with unique developmental and educational needs that differ significantly from those of younger children or young adults (Steinberg, 2014). To support learning in this developmental and educational stage, the Exact Path Middle School Skills Framework emphasizes leveraging students' growing cognitive capabilities, implementing instructional differentiation to meet diverse needs, and aligning teaching practices with rigorous, standards-based instruction that prepares students for future academic challenges. Immordino-Yang et al. (2019) stress that “adolescents' capacity for abstract and strategic thinking expands significantly during this developmental period, allowing them to engage with increasingly complex and interconnected ideas” (p. 192).

From a cognitive development perspective, students develop the capacity for abstract thinking skills during the middle school years, which enables them to engage in hypothetical reasoning, critical thinking, and exploration of complex moral and ethical questions (Wood, 2014). They begin to question previously



provide targeted support to prevent learning gaps from widening. By aligning instructional materials and curricula with state standards, these schools ensure consistency and coherence in learning experiences across grades and subjects and have a clear measure for looking at academic, grade-level growth. This approach supports vertical and horizontal alignment, allowing educators to build on prior knowledge while ensuring consistency across different classrooms and grade levels (Williams et al., 2010).

Effective intervention systems in high-performing middle schools enable educators to tailor instruction to meet individual student needs, ensuring that learning tasks are appropriately challenging and within the students' Zone of Proximal Development (ZPD), where they can achieve success with guidance and support. Through the use of standards-driven skills and focus on the ZPD, the Exact Path Middle School Skills Framework is the blueprint that supports a relevant and engaging pathway for content development. The ZPD is defined as the space between what a learner can do without assistance and what a learner can do with guidance or collaboration and was a main focus point for the thinking behind the development of the Exact Path Middle School Skills Framework. By creating just the right level of challenge and intentional support for remediation needs, Exact Path provides relevant and engaging learning for students.

In summary, addressing the learning needs of middle school students requires a multifaceted approach that combines rigorous academic standards with responsive, developmentally appropriate instruction. Programs that succeed in supporting this age group adopt data-driven practices, maintain consistent standards, and recognize the importance of differentiation and early intervention to bridge learning disparities—all elements of Exact Path's design. By providing a balanced approach that respects young adolescents' cognitive, social, and emotional needs, and cultivating a positive, inclusive school climate, educators targeting this age group can help students build the foundation for lifelong learning and success.

Edmentum's Exact Path Middle School Mathematics Skills Framework

Algebra Readiness

Because Algebra I is considered one of the most important gatekeeper courses in high school (Clotfelter et al., 2012), as it sets the foundation for future success in math and STEM fields, it is vital that middle school mathematics education prepare students for success in this course. Unfortunately, Algebra I failure rates are alarmingly high, particularly in urban schools, where students often struggle with many of the more abstract algebra concepts. Studies have shown that students who fail Algebra I are significantly less likely to graduate on time, with long-term consequences for their academic and career trajectories (Allensworth & Easton, 2007). National and international comparisons of student achievement indicate that it is between fourth and eighth grade when U.S. students in general, and historically marginalized and high-poverty students in particular, fall rapidly behind desired levels of



achievement (Carbonari et al., 2024). These students are more likely to struggle with algebra due to a lack of foundational math skills, limited access to resources, and other systemic barriers.

One factor contributing to these struggles is that students often enter high school lacking the problem-solving and metacognitive skills needed to engage meaningfully with algebraic reasoning. Schoenfeld (1992) emphasizes that mathematical success requires not only procedural fluency but also the ability to think critically, make sense of problems, and reflect on problem-solving strategies. This highlights the need for instructional approaches that support students in developing mathematical reasoning alongside content knowledge, ensuring they are equipped to tackle algebraic challenges effectively.

The COVID-19 pandemic worsened existing gaps in algebra readiness, particularly for low-income students. [The Hechinger Report](#) highlights how schools are grappling with the challenge of filling these gaps, with strategies such as “slowing down” instruction and providing more individualized support to help students recover lost learning. Programs aimed at addressing these gaps, such as tutoring and targeted interventions, are crucial but require sustained investment and innovation to be effective (Bhatt et al., 2024).

Algebra Readiness has become a major educational focus, and the Exact Path Middle School Skills Framework aims to ensure that students are not only meeting grade-level standards but also are equipped with the conceptual understanding required for success in Algebra I.

Key Decisions in Building the Exact Path Middle School Mathematics Skills Framework

The Exact Path Middle School Mathematics Skills Framework was informed by **five key decisions** aimed at making the content more engaging and effective for struggling middle school students:

1. Segmenting Lessons and Skills for Ideal Comprehension and Retention

Exact Path’s design prioritizes segmenting lessons into small, focused units to improve student comprehension and retention. This approach is firmly grounded in cognitive load theory (Perry et al., 2021), which emphasizes that learners process and retain information more effectively when it is presented in manageable chunks. By reducing the cognitive demands placed on working memory, this method helps students focus their attention on one concept at a time, facilitating deeper understanding.

Through careful segmentation and attention to cognitive load, Exact Path empowers students to achieve mastery without becoming overwhelmed, enabling them to progress confidently and effectively through increasingly complex material. In addition to focusing instruction on a single skill at a time, Exact Path lessons reinforce other cognitive load reduction strategies by using analogies and worked examples to help students transfer information from working memory to long-term memory without overwhelming their cognitive capacities (Maccini & Gagnon, 2000; McGinn et al., 2023).



Determine Ratios

What are some ratios that show the relationships of the instruments?

There are 6 trumpets and 2 trombones. The ratio of trumpets to trombones is 6 to 2. This ratio may also be written 6:2 or $\frac{6}{2}$.

This example compares two parts (trumpets and trombones) of the same whole (instruments). It is a [part-to-part ratio](#).

Another example of a part-to-part ratio is the ratio of trombones to saxophones, 2:5. You can also write this ratio 2 to 5 or $\frac{2}{5}$.

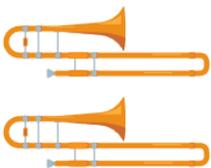
6 trumpets	2 trombones	5 saxophones	13 instruments
			$\begin{array}{r} 6 \text{ trumpets} \\ + 2 \text{ trombones} \\ + 5 \text{ saxophones} \\ \hline 13 \text{ instruments} \end{array}$

Figure 5. Grade 6 Math: Fractions and Ratios - Ratios and Ratio Language

In addition to narrowing the scope of content, Exact Path incorporates multiple modalities of instruction, such as combining verbal explanations with descriptive visuals. These multimodal strategies further support learning by reducing unnecessary cognitive demands and enhancing engagement. Research demonstrates that students retain information best when they process its meaning (Kang, 2016), which is why Exact Path integrates meaningful practice opportunities. By incorporating spaced practice layered with varied problem types, Exact Path reinforces learning over time, promoting long-term retention. Additionally, Exact Path's logical sequencing of content supports its segmented design, ensuring concepts are introduced in a structured and effective manner. Students learn by connecting new ideas to prior knowledge (Agodini et al., 2009), making well-sequenced curricula essential. By carefully connecting new concepts to prior knowledge, Exact Path ensures that students build on a strong foundation.

2. Building Proficiency in Rational Numbers Operations

Mastery of rational numbers, including fractions, decimals, and percentages, is critical for algebraic thinking. Exact Path prioritizes this skill to ensure students are prepared for Algebra I. Research by Joan Moss (2005) underscores the importance of rational number fluency for success in algebra learning. The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes the significance of number and operations as fundamental elements in the mathematics curriculum, underscoring their role in building a strong foundation for Algebra I. An intervention approach, specifically, must target properties of, and operations with, rational numbers as an essential prerequisite skill and building block for pre-algebra problem solving (Hurst & Cordes, 2017).

Powell & Gilbert (2019) concluded that one of the strongest predictors of algebra success is a solid understanding of rational numbers. Their findings indicate that students who struggle with fractions,



decimals, and ratios often encounter significant difficulties when transitioning to algebraic thinking. Encouraging students to move fluidly between different representations of rational numbers (fractions, decimals, and percentages) emphasizes the relationships between them, and this flexibility helps students make informed decisions on how to best approach problems involving rational numbers.

Summary: Ratios and Ratio Language
Watch this video for a summary of what you learned about writing and reading ratios and determining equivalent ratios.

Equivalent ratios

$$\frac{5}{2} \rightarrow \frac{5 \cdot 3}{2 \cdot 3} = \frac{15}{6}$$

5:2 is equivalent to 15:6.

0:22 / 0:58

transcript

Figure 6. Grade 6 Math: Fractions and Ratios - Ratios and Ratio Language

Many students rely on whole-number logic when encountering fractions and decimals, leading to significant misunderstandings. For example, students may incorrectly add numerators and denominators when adding fractions, rather than finding a common denominator, or they might interpret 0.35 as larger than 0.4, focusing on the number of digits rather than placement of the decimal point. To help students overcome these errors, Exact Path's supportive instruction allows students to visualize the relationships between fractions, decimals, and percentages through dynamic visual representations such as number lines and pie charts. By exposing students to interactive tools and engaging visual models, the program reinforces multiplicative reasoning and a deeper understanding of equivalency. Exact Path also integrates real-world applications and problem-solving tasks to help students apply rational number skills in meaningful contexts, thus deepening their comprehension and retention.

3. Investigating Proportionality, Proportional Reasoning, and Proportional Relationships

Proportional reasoning is the ability to recognize, understand, and work with relationships between quantities that maintain a consistent ratio, making it a critical predictor of success in algebra (Dole et al., 2015). This skill builds on the foundation of multiplicative reasoning, which involves grasping multiplication and division as operations and applying them to concepts such as ratios, scaling, and comparisons. While multiplicative reasoning focuses on performing calculations to determine how



quantities interact (e.g., doubling a recipe or dividing an amount equally), proportional reasoning takes this further by enabling students to identify patterns, generalize relationships, and apply them across various contexts.

Proportional reasoning skills enable students to transition from concrete arithmetic operations to abstract algebraic concepts such as variable manipulation, equations, and functions. Multiplicative reasoning provides the foundation for understanding ratios, which are at the heart of proportional reasoning. Students must first grasp multiplication and division as operations before they can reason about how quantities relate proportionally (Tzur et al., 2021).

Question

Type the correct answer in each box. Use numbers instead of words.
Use the steps to determine the unit rate for each given relationship.

Relationship	Original rate	Unit rate
Ashley uses 15 apples in 5 batches of apple-cinnamon bars. What is the rate of apples per batch?	$\frac{15 \text{ apples}}{5 \text{ batches}}$	$\frac{\div 5}{\div 5} = \frac{3 \text{ apples}}{1 \text{ batch}}$
Jenna drives 525 miles to the farmers market on 25 gallons of gas. What is her rate in miles per gallon?	$\frac{525 \text{ miles}}{25 \text{ gallons}}$	$\frac{\div 25}{\div 25} = \frac{21 \text{ miles}}{1 \text{ gallon}}$

Submit

Explanation:
Jenna drives 525 miles to the farmers market on 25 gallons of gas, so her original rate is $\frac{525 \text{ miles}}{25 \text{ gallons}}$. To find her unit rate, divide both terms by 25: $\frac{525 \text{ miles} \div 25}{25 \text{ gallons} \div 25} = \frac{21 \text{ miles}}{1 \text{ gallon}}$

Figure 7. Grade 6 Math: Fractions and Ratios – Unit Rates I

Students often struggle with proportional reasoning because traditional curricula treat arithmetic, fractions, and ratios as separate topics, limiting opportunities for students to see the connections between them. Chimoni & Pitta-Pantazi (2015) argue that proportional reasoning is a core component of algebraic thinking, as it allows students to develop the ability to analyze relationships between variables, a foundational skill for working with equations and functions. Their research suggests that students who build strong proportional reasoning skills are more adept at understanding algebraic structures and manipulating algebraic expressions. The Exact Path Middle School Skills Framework addresses this challenge by integrating instruction in ratios, fractions, and proportional relationships, ensuring students build these skills progressively and cohesively. By first developing a solid foundation in multiplicative reasoning, students are better equipped to understand proportional reasoning, which involves recognizing relationships between quantities, such as identifying that the ratio 3:6 is equivalent to 1:2. This understanding enables them to apply proportional reasoning to practical tasks, such as comparing unit prices or scaling recipes, fostering both conceptual understanding and real-world application.



The Difference Between Ratios, Rates, and Unit Rates

These are ratios because they are a comparison of two quantities with <i>the same units</i> .	These are rates because they have <i>different units</i> . Rates are a specific type of ratio.	These are unit rates because <i>the second quantity is equal to 1</i> . Unit rates are a specific type of ratio.
$\frac{4 \text{ pounds}}{2 \text{ pounds}}$ $\frac{\$8}{\$6}$ <p>4 apples : 3 apples 15 feet to 27 feet</p>	$\frac{\$6}{2 \text{ pounds}}$ $\frac{15 \text{ bulbs}}{\$3}$ <p>25 miles per 5 gallons 3 cups of flour to 4 cups of sugar</p>	$\frac{15 \text{ miles}}{1 \text{ hour}}$ $\frac{\$4}{1 \text{ bag}}$ $\frac{55 \text{ miles}}{\text{hour}}$ <p>20 miles per gallon</p>

Figure 8. Grade 6 Math: Fractions and Ratios – Unit Rates I

The framework prioritizes explicit instruction in ratios, fractions, and proportional relationships, which serve as the foundation for more complex mathematical concepts in high school. In Exact Path, students engage with tasks that move them from additive to multiplicative reasoning, applying these skills to real-world contexts such as interpreting scale maps, analyzing data in graphs, and determining the best value in product comparisons. These activities not only foster deeper conceptual understanding but also prepare students for the complexities of algebra and other higher-level math concepts. By embedding this instruction at multiple points along the framework, Exact Path ensures that students receive targeted practice and reinforcement, strengthening their ability to reason multiplicatively and bridging the gap between arithmetic and algebra.

4. Targeting Fluency in Arithmetic and Algebraic Operations

Fluency in arithmetic and algebraic computation is a cornerstone of Algebra I preparation and a key focus of the Exact Path Middle School Mathematics Skills Framework. Procedural fluency—the ability to perform mathematical operations swiftly and accurately—is essential for building confidence and competence in algebraic tasks (Hurst & Cordes, 2017). Research underscores the significance of both a strong understanding of magnitude in rational numbers and procedural knowledge, with arithmetic fluency emerging as a particularly strong predictor of algebra success (Dewolf et al., 2015). However, fluency is more than just speed; it requires efficiency, accuracy, and flexibility in mathematical thinking (Bay-Williams & San Giovanni, 2021). Students must develop multiple strategies for solving problems rather than relying solely on rote memorization, as flexible mathematical thinking enables deeper comprehension and the application of algebraic concepts. This finding suggests that developing students' flexibility with various notations and their ability to fluidly work with rational number representations not only supports algebraic thinking but also enhances students' overall mathematical achievement.

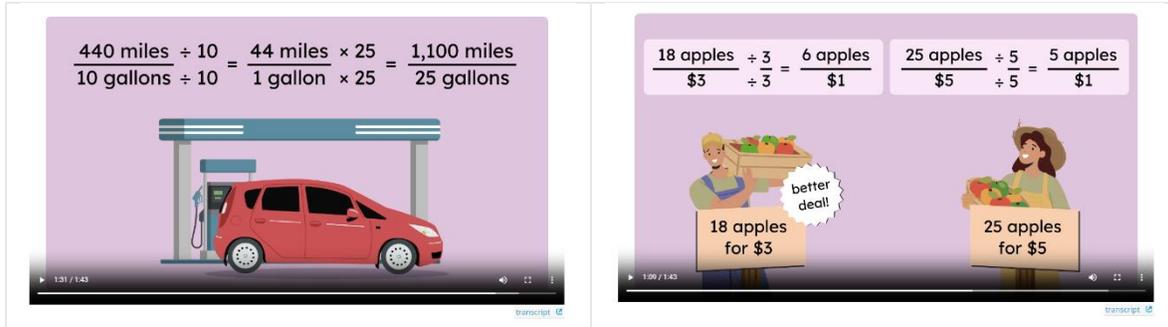


Figure 9. Grade 6 Math: Fractions and Ratios– Unit Rates I

To cultivate arithmetic and algebraic fluency, Exact Path combines targeted practice with interactive tools and real-world applications. Carefully sequenced lessons are designed to progressively build accuracy and speed in operations with fractions, decimals, and integers, ensuring students develop both procedural fluency and conceptual understanding.

For example, students practice operations with fractions and decimals in real-world contexts, such as scaling recipes or calculating discounts, to build accuracy and fluency. Interactive number lines, fraction bars, and area models are used to help students visualize and compare rational numbers, promoting a deeper understanding of their magnitudes. Additionally, Exact Path incorporates tasks in which students convert between fractions, decimals, and percentages, reinforcing their ability to fluently transition between different representations.

The Long Division Algorithm

The long division algorithm has five steps:

1. Divide
2. Multiply
3. Subtract
4. Bring down
5. Repeat

Play the video to see all the steps come together. For extra practice, pause the video at various points to see if you can predict what comes next.

The video has no audio.

0:33 / 0:38

transcript

Figure 10. Grade 6 Math: Numbers and Operations – Division of Whole Numbers

Beyond arithmetic fluency, Exact Path introduces algebraic operations in accessible, scaffolded steps. Students' progress from solving simple equations to working with multi-step problems involving variables and expressions. This comprehensive approach to fluency is further supported by deliberate integration of conceptual and procedural tasks. Students not only practice computations but also explore the reasoning behind the operations, fostering a deeper understanding of their applications in algebra. Through targeted and interactive practice, students not only strengthen their foundational skills but also build the algebraic thinking needed for future success.



5. Advancing from Conceptual Ideas to Concrete Practice

The Concrete-Representational-Abstract (CRA) model supports mathematical learning by helping students visualize and manipulate abstract concepts (Maccini & Gagnon, 2000). The model moves learners through the following steps/phases:

- **Concrete:** Students use physical objects (like blocks) to model math problems.
- **Representational:** Students draw or visualize these objects as pictures to understand the same problem.
- **Abstract:** Students use mathematical symbols and equations to solve the problem.

The CRA model also scaffolds learning for students who struggle with abstraction, providing gradual support as they transition from concrete experiences to more abstract mathematical thinking. Star et al. (2015) emphasize that successful algebra instruction requires making explicit connections between these representations, as students benefit from structured opportunities to move fluidly between concrete models, visual representations, and algebraic symbols. Furthermore, integrating multiple representations—such as number lines, area models, and algebra tiles—reinforces conceptual understanding and enhances procedural fluency. Research supports the effectiveness of this approach in improving comprehension and retention, particularly when teaching complex concepts such as variables, expressions, and functions (AL-salahat, 2022).

A structured progression, from concrete experiences to abstract representations, reinforces Exact Path's goal of fostering algebraic reasoning, which involves understanding and manipulating algebraic concepts, symbols, and structures. The Exact Path approach aligns with findings by Kaput et al. (2008), who advocate for integrating algebraic reasoning early in the curriculum and prioritizing conceptual understanding over rote procedural learning. For example, a student might begin by using virtual manipulatives to represent an unknown quantity, move to a diagram or visual representation of the problem, and ultimately solve the problem algebraically using variables and equations. By building strong connections between concrete, visual, and abstract representations, Exact Path ensures that students are equipped to construct and validate algebraic arguments, a critical skill for success in higher-level mathematics. By incorporating the CRA model, the Exact Path Middle School Mathematics Skills Framework empowers students to transition seamlessly from arithmetic to algebraic reasoning. This approach not only enhances mathematical literacy but also equips learners with critical thinking skills necessary for solving real-world problems and succeeding in advanced mathematics.

Research-Based Mathematics Skills Progression

Building on the foundational decisions that shaped the Exact Path Middle School Mathematics Skills Framework, it was essential to examine established frameworks and research that inform the development of a coherent and effective progression of skills. Algebra readiness is best developed through a structured and cumulative progression of skills, in which students gradually build numerical fluency, reasoning abilities, and problem-solving strategies necessary for success in Algebra I (Williams, 2011). This section explores the research-based foundations and guiding principles that underpin the Exact Path math skills progression, highlighting the logical sequence of concepts designed to reinforce understanding, build fluency, and prepare students for algebra and beyond.



One of the primary resources for the math skills progression was the National Council of Teachers of Mathematics (NCTM) which provides a comprehensive framework for improving mathematics education through several key publications, notably the [Principles and Standards for School Mathematics](#). This document lays out essential principles and standards to guide curricular decisions from pre-kindergarten through 12th grade. NCTM emphasizes a coherent, focused, and well-articulated curriculum that ensures a logical progression of concepts across grade levels. A central theme in NCTM's guidance is coherence across and within grade levels. This ensures that the curriculum is not merely a collection of disconnected activities but serves as a structured and purposeful sequence of learning experiences. The Exact Path Middle School Mathematics Skills Framework identifies key topics at each grade level, ensuring that students build on prior knowledge and progressively deepen their understanding of mathematical concepts. This coherence helps avoid fragmentation, promoting a more connected understanding of math, and aligns with NCTM recommendations.

Another critical resource for educators in creating a logical sequence of concepts is the [Achieve the Core](#) website, which identifies a subset of grade-specific math standards that lead to algebra, establishing a clear vertical alignment. This alignment fosters a seamless and cumulative learning experience for students, ensuring they are well-prepared for subsequent mathematical concepts. As part of a broad approach that incorporated multiple frameworks and state-specific standards, Achieve the Core and the Coherence Map provided useful insights into the progression of skills. The [Math Coherence Map](#) is an interactive tool that visually maps the connections between standards across grades, offering detailed tasks, instructional resources, and sample problems. These features help educators understand the progression and complexity of standards while facilitating coherent lesson development and were used in the Edmentum skills mapping process.

By comparing Edmentum curriculum maps with established frameworks (such as those provided by NCTM and Achieve the Core), LD was able to develop a more focused and cohesive curriculum. This approach not only aligns with key educational standards but also supports deeper student understanding and retention. The vertical alignment achieved through these resources ensures that students are progressively building their knowledge and skills, while maintaining a coherent and interconnected mathematical learning experience from year to year. For example, the topic of ratios and proportions is introduced in elementary grades as students learn to compare quantities and understand equivalent ratios. This foundational knowledge progresses to proportional relationships, when students apply these concepts to solve real-world problems. Eventually, it leads to the study of linear functions in algebra, when students analyze and model proportional relationships on a coordinate plane.

Referring to nationally recognized progression documents and resources was instrumental in shaping a curriculum that prepares students effectively for Algebra I and beyond, while ensuring a clear and purposeful learning trajectory.



The Exact Path Middle School ELA Skills Framework

Transitioning Beyond Foundational Literacy

NAEP data reveals that the majority of eighth grade students are “non-proficient” readers (NAEP, 2024). Many students enter high school underprepared, struggling with the demands of a curriculum that places greater emphasis on comprehension, textual analysis, and effective writing, while lacking proficiency in the foundational reading, writing, and analytical skills required for an effective transition (Gainer, 2012; Langer, 2001). Teachers must address these gaps while simultaneously advancing students’ abilities in reading comprehension, writing, and literary analysis to meet requirements for grade-level standards.

This transition, from foundational literacy to high school-level English, presents particular challenges for historically underserved populations, such as multilingual learners and students from low-income communities (Kieffer, 2008). These groups often face additional barriers, such as limited access to resources and language acquisition hurdles, further exacerbating gaps in reading and writing proficiency. As middle school teachers strive to bridge these gaps, they are tasked with developing students’ abilities in critical reading, analytical writing, and textual interpretation—essential skills for high school success. Focusing on explicit instruction in comprehension strategies, vocabulary development, and writing processes, Exact Path builds the foundation for students to engage critically with texts and articulate their understanding effectively. Additionally, scaffolded instruction ensures that students progress from basic literacy tasks to higher-order analytical skills, enabling them to meet the rigorous demands of high school English.

Preparing students for English 9 is a complex task that requires a combination of differentiated instruction, curriculum relevance, and targeted skill-building. By addressing diverse learning needs, integrating meaningful texts, and employing strategies that support comprehension and writing development, educators can better prepare students for the rigors of high school ELA. By fostering critical reading and writing abilities, the Exact Path Middle School ELA Skills Framework supports educators in preparing all students for the academic challenges ahead, regardless of their starting point.

Key Decisions in Building the Exact Path ELA Skills Framework

The Exact Path Middle School ELA Skills Framework was informed by **five key decisions focusing** on evidence-based literacy instruction aligned with the Science of Reading.

1. Filling Gaps for English 9 Readiness

Given the low rates of proficiency in eighth grade ELA skills (NAEP, 2024) and the expectations for high school ELA noted above, it is imperative that an intervention fill middle schoolers’ literacy gaps to ensure their readiness for the comprehension, analysis, and writing demands of English 9. To identify these critical gaps and skills, we analyzed the demands outlined in high school English standards, focusing specifically on the progression from middle school to high school expectations. Anchor Standards provided a broad understanding of what students need to achieve to be college and career ready, while grade-level standards helped pinpoint the discrete skills required by the end of eighth



grade to support ninth-grade success. Research studies, such as Torgesen et al. (2007) and Scammacca et al. (2007), provided insights into the literacy challenges faced by middle school students, particularly struggling readers. These studies emphasized foundational skills such as fluency and vocabulary, as well as higher-order skills such as comprehension strategies and critical thinking. Solis et al. (2018) found that targeted reading comprehension and vocabulary interventions significantly improve comprehension-related outcomes for students with low reading proficiency. Furthermore, this research underscores the importance of integrating explicit vocabulary instruction and scaffolded reading strategies to help struggling readers engage with complex texts. Using this background information in conjunction with research and data, LD created the Exact Path Middle School ELA Skills Framework to close gaps in the skills necessary for success in ninth-grade English through the selection of 30 focused skills at each grade level.

Specifically, the Exact Path Middle School ELA Skills Framework targets the following areas by creating clear skills statements aligned to standards to support content development rooted in pedagogically sound learning design. These areas include:

- **Fluency:** While reading rates may stabilize after sixth grade, students must continue to expand their vocabulary and word recognition to maintain fluency for the high school level texts they encounter in all content areas (Torgesen et al., 2007). The Exact Path Middle School ELA Skills Framework supports fluency development by providing explicit vocabulary instruction on a variety of skills including context clues and word analysis.
- **Vocabulary Knowledge:** This becomes increasingly critical as students encounter more complex texts. Explicit vocabulary instruction and exposure to new words in context are necessary (Moody et al., 2018). The Exact Path Middle School ELA Skills Framework supports vocabulary acquisition by providing instruction on how to determine the meanings of unknown words.
- **Higher-Level Thinking Skills:** Students need to develop the ability to draw inferences, make connections, and engage in critical thinking as they read more complex materials. (Rahmasari et al., 2024) The structure of the Exact Path Middle School ELA Skills Framework provides a progressive pathway to learning skills such as inference-making, theme, and author's perspective.
- **Reading Comprehension Strategies:** Explicit instruction in cognitive strategies, such as summarization, rereading, and questioning, helps students engage with and understand texts (Duke et al., 2002). The Exact Path Middle School ELA Skills Framework includes instruction and modeling within lessons that support students' use of strategies that can be employed in a variety of reading tasks.
- **Motivation and Engagement:** There is a strong relationship between motivation and reading proficiency, and motivation often declines in adolescence. Therefore, it is essential to implement strategies that actively engage students in reading and foster intrinsic motivation (Barber & Klauda, 2020). The structure of the Exact Path Middle School ELA Skills Framework encourages student engagement by ensuring that students receive the instruction they need



when they need it. The content within a skill is crafted to be engaging by being grounded in real-world examples and texts that are age-appropriate and connected to student experiences.

2. Skill Progression Across Grade Bands

Chall's (1983) stages of literacy development emphasize the transition from "learning to read" to "reading to learn," which typically occurs in about fourth grade. The first stage, "learning to read," focuses on acquiring foundational literacy skills such as phonemic awareness, phonics, fluency, vocabulary, and basic comprehension, as learners master the mechanics of reading. The second stage, "reading to learn," extends into high school, and thus was vital to designing the Exact Path Middle School ELA Skills Framework. This stage emphasizes the shift from decoding text to using reading as a tool for learning across content areas, developing critical thinking, and expanding academic vocabulary. This transition underscores the importance of designing literacy instruction that evolves to meet the changing demands of learners across grade levels.

Compare and Contrast Nonfiction: Pandas
Let's compare and contrast the authors' approaches in the texts about pandas.
This table shows one similarity and some of the differences.

Panda Text Excerpts		
Text 1	Both texts	Text 2
<ul style="list-style-type: none">Informational textInforms reader about pandas' diet, habitat, and behaviors	<ul style="list-style-type: none">Pandas	<ul style="list-style-type: none">Persuasive textUses facts to support argument that pandas have many challenges

Question
Reread the text excerpts about pandas. Then select the correct answer.
How does the author's purpose in text 2 differ from their purpose in text 1?

- The author of text 2 is encouraging people to watch pandas play.
- The author of text 2 is encouraging people to help protect pandas.
- The author of text 2 is discouraging people from opening panda preserves.
- The author of text 2 is discouraging people from giving pandas bamboo.

Submit

Explanation:
In text 2 the author presents the argument that the pandas' "biggest threat is habitat loss," with the purpose of encouraging people to help protect pandas. This approach is different from that of text 1, where the author gives information about pandas, with the purpose of describing pandas' appearance and behavior.

Figure 11. Grade 6 Reading: Reading Informational Texts – Compare and Contrast Nonfiction Texts

Research consistently supports the need for a strong foundation in early literacy skills to facilitate advanced comprehension and analysis in later years. Explicit instruction in comprehension strategies, as advocated by Block and Parris (2008), is essential during the "reading to learn" phase. These strategies include summarization, inference-making, and questioning, which are critical for helping students engage with increasingly complex texts. However, while research provides a clear sequence for foundational literacy development, there is less consensus on a systematic progression for higher-order literacy skills, such as advanced comprehension, analysis, and synthesis. The Council of Writing Program Administrators, National Council of Teachers of English, and National Writing Project (2011) emphasizes the importance of integrating these skills into a coherent, cross-grade progression that builds on prior knowledge while promoting deeper understanding in new contexts.

The Exact Path Middle School ELA Skills Framework was intentionally designed to promote continuity in learning without unnecessary repetition. By leveraging prior knowledge, students are challenged to expand their capabilities through tasks that align with their developmental and academic readiness. For instance, comprehension strategies are scaffolded from identifying main ideas and supporting details in elementary grades to analyzing themes, evaluating arguments, and synthesizing information across multiple texts by middle school. This approach ensures that students deepen their understanding and apply skills in more complex contexts, fostering both confidence and competence.



3. Standards Alignment

Research consistently highlights the value of aligning curricula with standards, not only in enhancing teaching effectiveness but also in fostering student success. Reports have emphasized that standards-aligned curriculum materials, when accompanied by appropriate instructional support, significantly improve learning outcomes. Borman et al. (2008) examined the effects of specific reading and math curricula aligned with state standards on student achievement and found that students who were taught using standards-aligned curriculum materials showed significant improvements in their academic performance compared to those who did not use such materials. The connection between standards-aligned resources and student achievement is further strengthened when teachers receive targeted professional development to implement these materials effectively. Therefore, Edmentum focuses not only on ensuring that the content adheres to current standards but also on supporting Exact Path's successful implementation in the classroom.

Standards coverage played a critical role in the development of the Exact Path Middle School ELA Skills Framework, ensuring a comprehensive alignment that prepares students for high school. The framework emphasizes essential skills foundational for English 9 readiness while incorporating benchmarks and standards that states use to meet diverse educational requirements. This careful design ensures that the framework supports a broad and rigorous ELA curriculum adaptable to different state and district expectations.

To support effective implementation, Exact Path includes teacher resources such as lesson ideas, standards alignment charts, and professional development tools. The lesson ideas offer step-by-step instructions for delivering additional content effectively, with suggested pacing and strategies to differentiate instruction for diverse learners. Meanwhile, standards alignment charts help educators clearly identify how each lesson addresses relevant standards, ensuring that instruction remains focused, and standards driven. These resources help educators understand how to use the curriculum to meet standards while adapting to the specific needs of their students.

4. Science of Reading: Teaching Reading Explicitly and Systematically

In 2000, the National Reading Panel released a report that outlined the most effective instructional approaches to teach students to read. These instructional approaches became known as the Science of Reading. This body of research-backed best practices is divided into five pillars: phonological awareness, phonics, fluency, vocabulary, and comprehension.

The complexities of written language demand deliberate and structured teaching to ensure success. This focus on structure and explicit teaching became a focal point for the Exact Path Middle School ELA Skills Framework. Research consistently highlights direct, explicit, and systematic instruction as essential strategies for teaching both foundational and advanced literacy skills. These instructional practices are firmly rooted in decades of evidence-based research, including the findings of the *National Reading Panel Report* (NICHD, 2000). Denton et al. (2003) affirm the impact of this approach, noting that "explicit instruction in reading makes a difference in student outcomes, especially for those who are low achieving" (p. 202). Explicit teaching ensures clarity by breaking down concepts and processes step by step. Their research also emphasizes that explicit instruction has consistently

improved outcomes in phonological awareness, decoding, and comprehension strategies, demonstrating its efficacy in supporting struggling readers.



Figure 12. Grade 6 Reading: Language and Vocabulary – Roots and Affixes

Chall and Jacobs (2003) emphasize the need for explicit vocabulary instruction in developing comprehension, particularly as students encounter advanced academic language in middle school. Rooted in the Science of Reading, this approach integrates foundational skills such as decoding and fluency with higher-order strategies such as critical analysis and synthesis. Vocabulary acquisition plays a crucial role in comprehension, enabling students to access and engage with complex texts (Diamond & Gutlohn, 2006). Explicit vocabulary instruction ensures that students not only recognize words but also understand their meanings and contexts, laying a foundation for comprehension. Exact Path aligns with these principles by incorporating structured vocabulary instruction across units, focusing on both domain-specific and general academic vocabulary. Beck et al. (2002) found that direct vocabulary instruction significantly enhances comprehension, especially for informational texts. By embedding vocabulary work within its framework, Exact Path equips students with the tools needed to tackle increasingly complex texts.

Systematic instruction further complements this approach by following a logical, pre-planned sequence that builds on prior learning. As Adams (2001) explains, “the goal of systematic instruction is one of maximizing the likelihood that whenever children are asked to learn something new, they already possess the appropriate prior knowledge and understandings to see its value and to learn it efficiently” (p. 74). By teaching skills progressively, moving from simple to complex, systematic instruction ensures that each component of literacy is taught in a structured and connected way. This



method is particularly effective for preventing reading difficulties, as Walpole et al. (2004) highlight, noting the importance of “targeting such skills explicitly and directly” (p. 265).

By middle school, most students have established a foundation in phonological awareness and phonics. At this stage, the instructional focus shifts toward comprehension, vocabulary, and fluency using increasingly complex texts. The Exact Path Middle School ELA Skills Framework provides a systematic roadmap of standards aligned skills that leverages this research by embedding direct, explicit, and systematic instruction into its literacy curriculum with a focus on middle school needs. Each Exact Path lesson begins with clear, explicit explanations of skills and concepts, providing students with step-by-step modeling to ensure understanding. Systematic instruction ensures skills are taught in a logical sequence, moving from foundational to advanced literacy tasks, while building on prior knowledge. For example, comprehension strategies such as summarizing and inference-making are introduced through direct teaching, then reinforced with guided and independent practice using increasingly complex texts. Exact Path also integrates scaffolding to support students at every stage of learning, gradually releasing responsibility as they gain proficiency. By aligning with the principles of explicit and systematic instruction, Exact Path ensures that students develop the skills necessary to navigate challenging texts, fostering both confidence and long-term literacy success

5. Evidence-Based Literacy Instruction

The Exact Path Middle School ELA Skills Framework is grounded in the Active View of Reading Model (Duke & Cartwright, 2021), a contemporary evolution of the widely recognized Simple View of Reading (SVR; Gough & Tunmer, 1986). The SVR conceptualizes reading as the product of decoding (D) and language comprehension (C), expressed as $D \times C = R$ (reading). While the SVR remains foundational in understanding the Science of Reading, the Active View of Reading builds on it to reflect a more nuanced understanding of literacy development. By incorporating Scarborough’s Reading Rope (2001), the Active View of Reading emphasizes the interconnected processes of word recognition and language comprehension, where both components are made up of braided strands of skills that work together to produce skilled, strategic reading (See Figure 1).

Scarborough’s Reading Rope expanded the SVR equation by detailing the subskills required to decode and comprehend. Word recognition includes phonological awareness, decoding, spelling, and sight word recognition, while language comprehension encompasses background knowledge, vocabulary, language structures, verbal reasoning, and literacy knowledge. As students’ progress, the Reading Rope illustrates how word recognition becomes increasingly automatic, freeing cognitive resources for strategic, deeper comprehension of text. This model continues to underpin Exact Path’s foundational reading instruction, which is explicitly designed to support students in developing both the automaticity and cognitive strategies necessary for skilled reading.

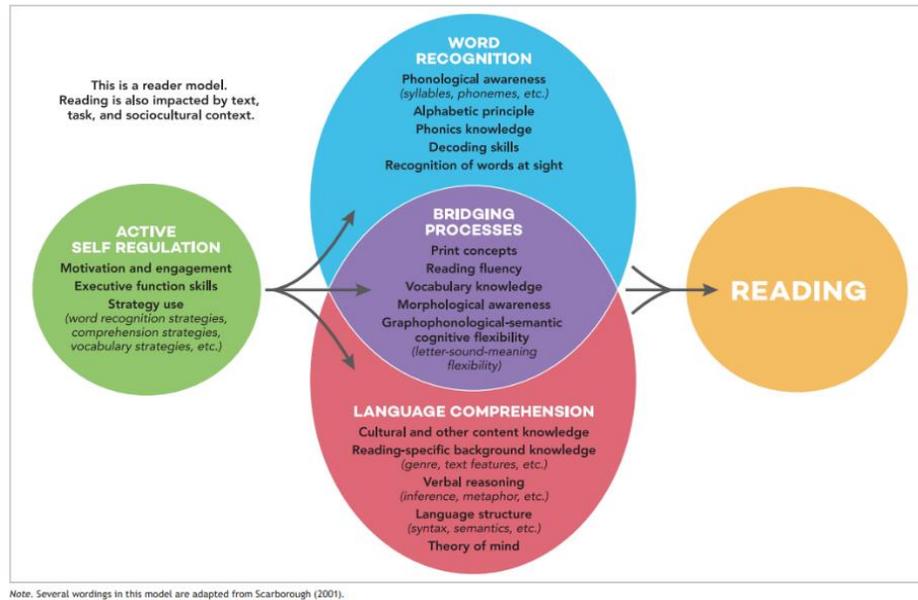


Figure 12. Active View of Reading Model (Duke & Cartwright, 2021, p. S33)

A significant advancement that Duke and Cartwright (2021) offer in their Active View of Reading Model is the recognition of executive function (EF) as a critical element in reading development. They identify EF skills—such as cognitive flexibility, working memory, inhibitory control, planning, and attentional regulation—as essential tools that skilled readers use to “make reading happen” (p. S30). For instance, readers must switch flexibly between decoding words and deriving meaning (graphophonological-semantic cognitive flexibility) or hold key information in working memory to connect ideas across paragraphs. These cognitive processes enable readers to manage the complex demands of reading, particularly as they engage with more challenging texts. Duke and Cartwright further argue that motivation and engagement play pivotal roles in active, self-regulated reading, directly influencing literacy outcomes beyond word recognition and comprehension alone.

Exact Path’s Middle School ELA Skills Framework translates these research-based insights into a coherent instructional approach. The curriculum integrates the foundational literacy components identified in the Science of Reading—such as phonemic awareness, fluency, and vocabulary—while systematically building toward higher-order skills such as critical analysis, synthesis, and evaluation. Exact Path also emphasizes the interconnected nature of reading and language arts skills by teaching students to analyze sentence structures, evaluate how authors develop their arguments, and expand vocabulary through context-driven text analysis. These skills are explicitly taught and scaffolded to ensure students can navigate complex texts with confidence and independence.

To deepen comprehension, Exact Path incorporates cognitive strategies such as summarization, questioning, inference-making, and self-monitoring. These strategies are introduced with explicit instruction and reinforced through meaningful, real-world applications. For example, students might analyze informational texts to understand how authors support claims with evidence or engage in close reading of literature to infer themes and symbolism. By fostering these connections between cognitive



strategies, reading skills, and language arts outcomes, Exact Path provides students with the tools to engage actively with texts and think critically about their content.

Furthermore, Exact Path's focus on executive function ensures that instruction not only builds literacy skills but also supports broader cognitive development. Activities are designed to strengthen working memory, attentional control, and cognitive flexibility, enabling students to manage the increasing demands of academic texts. Coupled with an emphasis on motivation and engagement, this approach empowers students to become active, self-regulated readers who can apply their skills in meaningful and authentic contexts.

Research-Based ELA Skills Progression

The research around ELA learning highlights the importance of building skills progressively to support vocabulary development and comprehension.

The most compelling structure for driving the Exact Path Middle School ELA Skills Framework is standards alignment. National and state standards provide a clear progression of learning goals that flow from one grade to the next.

- Anchor standards act as an umbrella, providing a big picture view of what students should know and be able to do in order to be college and career ready.
- Grade-level standards offer additional specificity, outlining the discrete skills students need to know and be able to do by the end of each grade.

The vertical progression of standards from grade-to-grade act as a bridge between the anchor and grade-level standards, showing us how a given standard evolves over time and extends previous learning.

Another concept that guided the development of the Exact Path Middle School ELA Skills Framework is the importance of designing for recursive instruction. Core Knowledge Foundation (2023) stresses the importance of sequencing within grades to build complexity over time, reinforce key skills, and maintain developmental appropriateness. A recursive approach revisits and refines critical skills throughout the year, allowing students to deepen their mastery. For example, reading comprehension strategies such as inferring and summarizing are revisited across units and contexts, ensuring consistent reinforcement (Guthrie & Wigfield, 2000).

LD also considered the need to gradually increase text complexity when designing the framework. Simpler texts with linear narratives are used at the start of the year, gradually transitioning to more sophisticated texts with nonlinear timelines or multiple points of view. This approach scaffolds students' analytical skills, helping them move from basic literary analysis, such as identifying setting and character, to more nuanced tasks such as interpreting symbolism or comparing themes across texts (Chall & Jacobs, 2003). This intentional sequencing not only ensures the development of essential ELA skills but also promotes students' retention and application in increasingly challenging contexts throughout the academic year.

Exact Path employs a standards-aligned, recursive, progressively structured approach in its Middle School ELA Skills Framework. Early in the year, foundational skills such as identifying main ideas and



summarizing are introduced, providing a base for more complex tasks later, such as analyzing themes or evaluating arguments. This structured progression aligns with students' cognitive growth, fostering both confidence and competence (Piaget, 1964).

Conclusion

The Exact Path Middle School Skills Framework is a comprehensive, research-backed solution designed to address the critical challenges facing middle school students in both mathematics and ELA. By focusing on key skills, aligning with educational standards, and incorporating research-backed instructional approaches, the framework ensures that students are well-prepared for the challenges of high school. With its foundation in evidence-based practices and feedback from educators, this framework represents a significant step forward in middle school intervention.



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